# Supplemental Information – Photon Blockade in an Optical Cavity with One Trapped Atom

K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble
*Norman Bridge Laboratory of Physics 12-33*
*California Institute of Technology, Pasadena, CA 91125*

In our Letter [1], we employ a model of a two mode cavity coupled to an atom with multiple internal states. In this Supplement we make explicit the coupling used in the model Hamiltonian to determine the eigenvalues displayed in Figure 1(b) of Ref. [1]. This Hamiltonian was also incorporated into the master equation for the damped, driven system used to compute the theoretical results in Fig. 2(b) of Ref. [1]. We furthermore present an extension to this model which includes the effect of cavity birefringence and FORT induced ac-Stark shifts in the atomic states. The modified cavity transmission and intensity correlation function are presented for comparison.

We approximate the atom-cavity coupling to be a dipole interaction. We define the atomic dipole transition operators for the $6S_{1/2}, F = 4 \to 6P_{3/2}, F' = 5'$ transition in atomic Cæsium as

$$D_q = \sum_{m_F=-4}^{4} |F = 4, m_F\rangle\langle F = 4, m_F|\mu_q|F' = 5', m_F + q\rangle\langle F' = 5', m_F + q| \ , \tag{1}$$

where $q = \{-1, 0, 1\}$ and $\mu_q$ is the dipole operator for $\{\sigma_-, \pi, \sigma_+\}$-polarization, respectively, normalized such that for the cycling transition $\langle F = 4, m_F = 4|\mu_1|F' = 5', m_F = 5\rangle = 1$. The matrix element of the dipole operator $\langle F = 4, m_F|\mu_q|F' = 5', m'_F\rangle$ is equivalent to the Clebsch-Gordan coefficient for adding spin 1 to spin 4 to reach total spin 5, namely $\langle j_1 = 4, j_2 = 1; m_1 = m_F, m_2 = q|j_{total} = 5; m_{total} = m'_F\rangle$.

The Hamiltonian of a single atom coupled to a cavity with two degenerate orthogonal linear modes is

$$\begin{aligned} H_{4\to5'} \ &= \ \hbar\omega_A \sum_{m'_F=-5}^{5} |F' = 5', m'_F\rangle\langle F' = 5', m'_F| + \hbar\omega_{C_1}(a^\dagger a + b^\dagger b) \\ &\quad +\hbar g_0(a^\dagger D_0 + D_0^\dagger a + b^\dagger D_y + D_y^\dagger b) \ , \end{aligned} \tag{2}$$

where $D_y = \frac{i}{\sqrt{2}}(D_{-1} + D_{+1})$ is the dipole operator for linear polarization along the $y$-axis. We are using coordinates where the cavity supports $\hat{y}$ and $\hat{z}$ polarizations and $\hat{x}$ is along the cavity axis. The annihilation operator for the $\hat{z}$ ($\hat{y}$) polarized cavity mode is $a$ ($b$).

Assuming $\omega_A = \omega_{C_1} \equiv \omega_0$, we find that the lowest eigenvalues of $H_{4\to5'}$ have a relatively simple structure. In the manifold of zero excitations all nine eigenvalues are zero. In manifolds with $n$ excitations, the eigenvalues are of the form $E_{n,k} = n\hbar\omega_0 + \hbar g_0 \varepsilon_k^{(n)}$, where $\varepsilon_k^{(n)}$ is a numerical factor and $k$ is an index for distinct eigenvalues. There are 29 states in the $n = 1$ manifold, but due to degeneracy $k$ has only 13 distinct values, $k \in \{-6, \ldots 6\}$; in the $n = 2$ manifold there are 49 states but $k \in \{-11, \ldots 11\}$. The number of states in any manifold can be understood by considering how the excitations can be distributed among the atom and the two cavity modes. For example, in the $n = 1$ manifold, the atom can be in one of its 9 ground states ($m_F \in \{-4, \ldots 4\}$) and either cavity mode $l_y$ or mode $l_z$ can have one photon (giving 18 possible states), or the atom can be in one of its 11 excited states ($m'_F \in \{-5, \ldots 5\}$) while both cavity modes are in the vacuum state, yielding a total of 29 states. Table I lists numerical values for $\varepsilon_k^{(1,2)}$ as well as their respective degeneracies $\eta_k^{(1,2)}$. The numerical factors and degeneracies have the symmetries $\varepsilon_{-k}^{(n)} = -\varepsilon_k^{(n)}$ and $\eta_{-k}^{(n)} = \eta_k^{(n)}$. The resulting eigenvalues $E_{n,k}$ for $n = \{0, 1, 2\}$ are displayed in Fig. 1(b) of Ref. [1].

Although these eigenvalues are certainly not sufficient for understanding the complex dynamics associated with the full master equation, they do provide some insight into some structural aspects of the atom-cavity system. For example, the eigenvalues $\varepsilon_{\pm6}^{(1)} = \pm1$ correspond to the vacuum-Rabi splitting for the states $|1, \pm\rangle$ for a two-state atom coupled to a single cavity mode [cf., Fig. 1(a) in Ref. [1]]. The one-photon detunings for transitions from the $n = 1 \to n' = 2$ manifold are largest for the eigenstates associated with $\varepsilon_{\pm6}^{(1)}$. Indeed, just as for the two-state atom with one cavity mode, transitions from the eigenstates at $\pm g_0$ have frequency detunings $\pm(2 - \sqrt{2})g_0$ relative to the nearest states in the $n' = 2$ manifold (at $\varepsilon_{\pm11}^{(2)} = \pm\sqrt{2}$, respectively). Hence, as a function of probe frequency $\omega_p$, the eigenvalue structure in Table I suggests that the ratio of two-photon to one-photon excitation would exhibit a minimum around $\omega_p = \omega_0 \pm g_0$, resulting in reduced values $g^{(2)}(0) < 1$, which the full calculation verifies in Fig. 2(b) of Ref. [1].

| k | $\varepsilon_k^{(1)}$ | $\eta_k^{(1)}$ | $\varepsilon_k^{(2)}$ | $\eta_k^{(2)}$ |
|---|---|---|---|---|
| 0 | 0 | 7 | 0 | 5 |
| 1 | 0.667 | 1 | 0.516 | 1 |
| 2 | 0.683 | 2 | 0.556 | 2 |
| 3 | 0.730 | 2 | 0.662 | 2 |
| 4 | 0.803 | 2 | 0.805 | 2 |
| 5 | 0.894 | 2 | 0.966 | 3 |
| 6 | 1 | 2 | 0.978 | 2 |
| 7 | | | 1.014 | 2 |
| 8 | | | 1.073 | 2 |
| 9 | | | 1.155 | 2 |
| 10 | | | 1.265 | 2 |
| 11 | | | 1.414 | 2 |

TABLE I: Numerical factors $\varepsilon_k^{(n)}$ for the eigenvalues of the Hamiltonian $H_{4\rightarrow 5'}$ in Eq. 2, together with their degeneracies $\eta_k^{(n)}$.

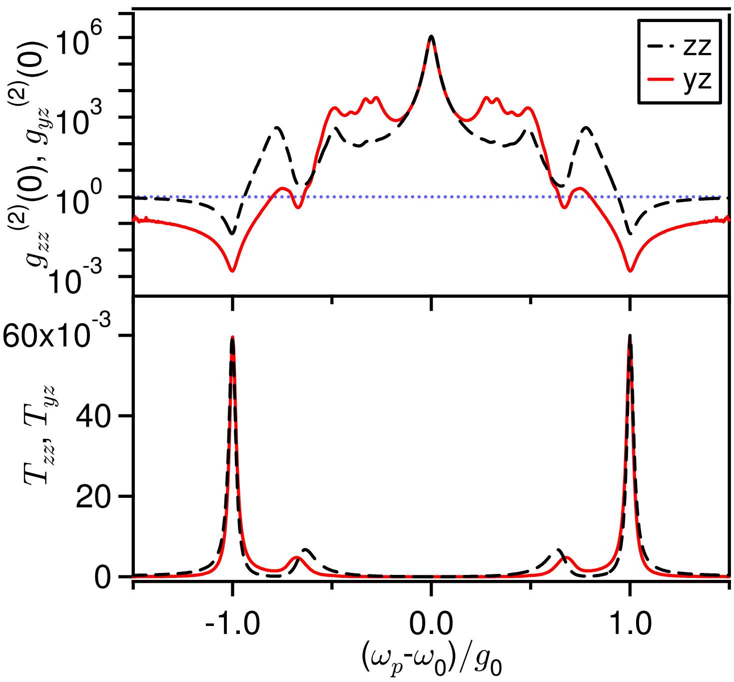


FIG. 1: $T_{zz}$ and $g_{zz}^{(2)}(0)$ (dashed), and $T_{yz}$ and $g_{yz}^{(2)}(0)$ (red) versus normalized probe detuning (see Ref. [1] for definition of variables). We consider an $F = 4 \rightarrow F' = 5'$ transition driven by linearly polarized light in a cavity containing two modes of orthogonal polarization that are frequency degenerate. Parameters are $(g_0, \kappa, \gamma)/2\pi = (50, 1, 1)$ MHz. The probe strength is such that the intracavity photon number on resonance without an atom is 0.05. The blue dotted line indicates $g^{(2)}(0) = 1$ for Poissonian statistics.

For excitation to the other eigenstates in the $n = 1$ manifold, such blockade is not evidenced in Fig. 2(b) [1]. A contributing factor suggested by the structure of eigenvalues in Table I is interference of one and two-photon excitation processes. For example, excitation at $\omega_p \simeq \omega_0 \pm g_0/4$ results in two-photon resonance for the eigenstates associated with $\varepsilon_{\pm 1}^{(2)} \simeq \pm 0.5$, and to photon bunching with $g^{(2)}(0) \gg 1$ as confirmed by our full calculation of photon statistics.

Figure 1 provides a global perspective of these various effects. Here, we calculate transmission spectra and intensity correlation functions analogous to those shown in Fig. 2(b) of Ref. [1], but now with coherent coupling $g_0$ much larger than the dissipative rates $(\kappa, \gamma)$ and well beyond what we have achieved in our experiments, $g_0/\kappa = g_0/\gamma = 50$. At $\omega_p = \omega_0 \pm g_0$, $g_{yz}^{(2)}(0) \simeq 0.002$ in evidence of the previously discussed photon blockade suggested by the eigenvalue structure in Table I. As anticipated, large photon bunching results near $\omega_p \simeq \omega_0 \pm g_0/4$ associated with the two-photon

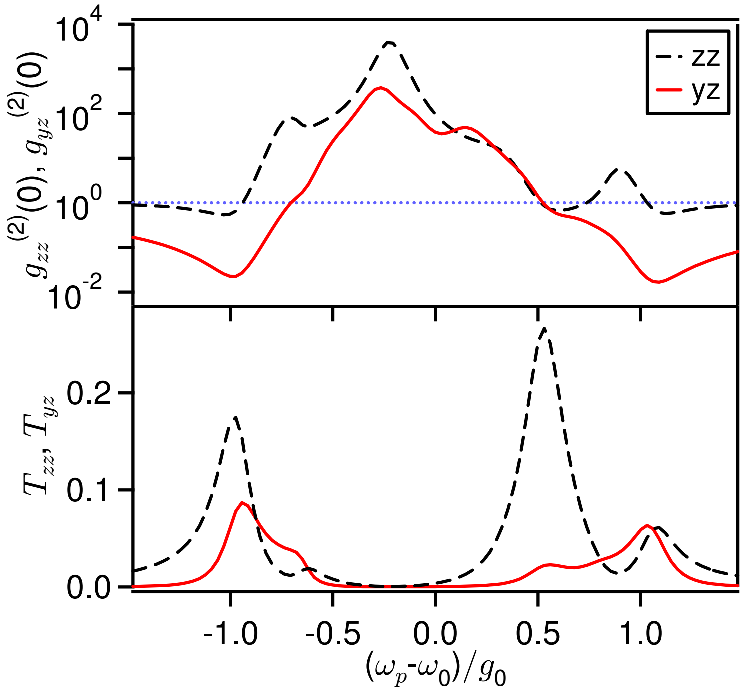


FIG. 2: $T_{zz}$ and $g_{zz}^{(2)}(0)$ (dashed), and $T_{yz}$ and $g_{yz}^{(2)}(0)$ (red) versus normalized probe detuning (see Ref. [1] for definition of variables). We consider an $F = 4 \to F' = 5'$ transition driven by linearly polarized light in a cavity containing two nondegenerate modes of orthogonal polarization. Parameters are $(g_0, \kappa, \gamma, \Delta\omega_{C_1}, U_0)/2\pi = (33.9, 4.1, 2.6, 4.4, -43)$ MHz, and $\omega_{C_1^z} = \omega_A \equiv \omega_0$. The probe strength is such that the intracavity photon number on resonance without an atom is 0.05. The blue dotted line indicates $g^{(2)}(0) = 1$ for Poissonian statistics.

resonance to reach the eigenstates with $\varepsilon_{\pm 1}^{(2)} \simeq \pm 0.5$. Between these two extremes for the eigenvalues with the largest and smallest nonzero magnitudes ($\omega_0 \pm g_0 \leq \omega_p \leq \omega_0 \pm g_0/4$), $g_{yz}^{(2)}(0)$ displays a complex structure involving multiple excitation pathways through states in the $n = 1$ manifold to reach states in the $n' = 2$ manifold. The extremely large peak at $\omega_p = \omega_0$ is discussed in Refs. [2, 3].

We now consider the effects of cavity birefringence and $m'_F$-dependent ac-Stark shifts. These modify the Hamiltonian $H_{4\to 5'}$ in Eq. 2 to

$$\begin{aligned} H_{full} &= \sum_{m'_F=-5}^{5} \hbar\omega_{m'_F} |F' = 5', m'_F\rangle\langle F' = 5', m'_F| + \hbar\omega_{C_1^z} a^\dagger a + \hbar\omega_{C_1^y} b^\dagger b \\ &\quad + \hbar g_0 (a^\dagger D_0 + D_0^\dagger a + b^\dagger D_y + D_y^\dagger b) \end{aligned} \tag{3}$$

The birefringent splitting $\Delta\omega_{C_1}$ is the difference of the resonant frequencies of the two polarization modes, $\Delta\omega_{C_1} = \omega_{C_1^z} - \omega_{C_1^y}$. The atomic excited state frequencies are given by $\omega_{m'_F} = \omega_A + U_0 \beta_{m'_F}$, where $\omega_A$ is the unshifted frequency of the $F = 4 \to F' = 5'$ transition in free space, $U_0$ is the FORT potential, and $\beta_{m'_F}$ for the FORT wavelength of the experiment is given by $\{m'_F, \beta_{m'_F}\} = \{\pm 5, 0.18\}, \{\pm 4, 0.06\}, \{\pm 3, -0.03\}, \{\pm 2, -0.10\}, \{\pm 1, -0.14\}, \{0, -0.15\}$ [4].

The effect of these corrections to the Hamiltonian on the transmitted field from the steady-state solutions to the master equation are displayed in Fig. 2. The heights and shapes of the multiplets in $T_{yz,zz}$ are modified, but the basic structure is unaffected relative to Fig. 2(b) of Ref. [1]. The structure of $g_{yz,zz}^{(2)}(0)$ is also qualitatively unchanged. The value of $g_{yz}^{(2)}(0)$ for $\omega_p = \omega_0 - g_0$ is 0.02 (ignoring the above corrections yields $g_{yz}^{(2)}(0) \simeq 0.03$). These values are consistent with the experimental result $g_{yz}^{(2)}(0) = 0.13 \pm 0.11$ [1].

[1] Birnbaum, K. M., Boca, A., Miller, R., Boozer, A. D., Northup, T. E., and Kimble, H. J. Photon Blockade in an Optical Cavity with One Trapped Atom (2005).

[2] Carmichael, H. J, Brecha, R. J., & Rice, P. R. Quantum interference and collapse of the wavefunction in cavity QED. *Opt. Commun.* **82**, 73-79 (1991).

[3] Brecha, R. J., Rice, P. R., and Xiao, M. N two-level atoms in a driven optical cavity: Quantum dynamics of forward photon scattering for weak incident fields. *Phys. Rev. A* **59**, 2392-2417 (1999).

[4] McKeever, J., *et al.* State-Insensitive Cooling and Trapping of Single Atoms in an Optical Cavity. *Phys. Rev. Lett.* **90** , 133602 (2003).